MYP MAY 2016

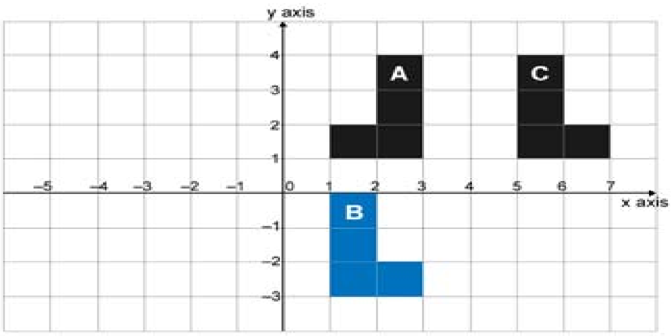
EXTENDED MATHEMATICS

ON-SCREEN EXAMINATION

Exemplar Marked Candidate Responses

This document contains exemplar material which demonstrates how the markscheme was applied to two student responses for the May 2016 session. Teachers should consider the application of the marksheme and in particular the assessment of longer, open ended responses. Teachers may wish to mark the student response themselves using the published markscheme and then compare their marking to the standard demonstrated in this document.

Question 1 (6 marks)



Question 1a (1 mark)

Write down the equation of the line that reflects shape A to shape C.

x=4 1/1

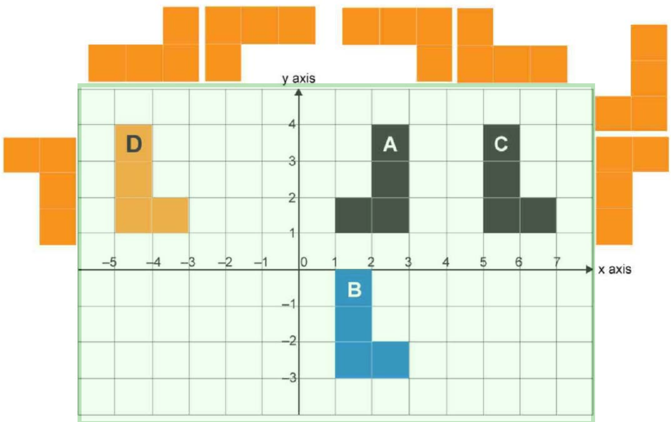
Question 1b (1 mark)

The vector ***c*** translates the shape B to shape C. **Write down** the vector ***c*** in the form $\begin{pmatrix} p \\ q \end{pmatrix}$.

$\begin{pmatrix} 4 \\ 4 \end{pmatrix}$ 1/1

Question 1c (1 mark)

Translate shape B by ve $\begin{pmatrix} -4 \\ 4 \end{pmatrix}$, **draw** and label the image D.



0/1

Question 1d (2 marks)

Find the scalar product ***c* · *d*** .

$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ 0/2

Question 1e (1 mark)

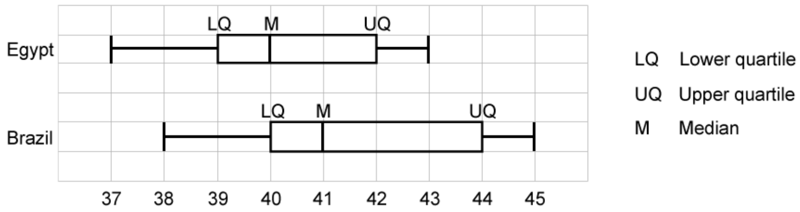
Explain the significance of your result in part (d).

$\left|\begin{pmatrix} 5 \\ 1 \end{pmatrix}\right|\left|\begin{pmatrix} 5 \\ 1 \end{pmatrix}\right|\cos(180)$ 0/1

Question 2 (8 marks)

A manufacturer makes shoes to sell internationally. The manufacturer is contracted to make shoes for Egypt and Brazil.

The box plots show the sizes of shoes sold in Egypt and Brazil. The distributions shown in the box plots are based on the medians and quartiles of the shoe sizes sold.



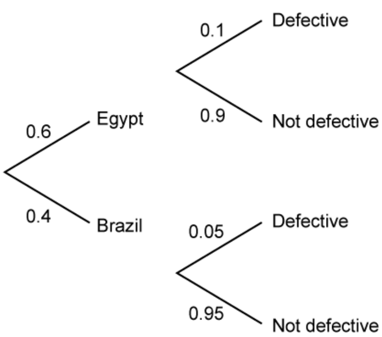
Question 2a (2 marks)

Compare the sizes of shoes sold in the two countries.

The shoes sold in Egypt range from the sizes 37-43, and the shoes sold in Brazil range from the sizes 38-45.

0/2 Range with values but no comment.

The manufacturer makes shoes in two factories; one in Egypt and one in Brazil. The factory in Egypt makes 60 % of the shoes and 10 % are defective. The factory in Brazil makes the rest of the shoes and 5 % are defective.



Question 2b (3 marks)

Find the probability that a shoe, chosen at random from either factory, is defective.

(0.6*0.1)+(0.4*0.05)
=0.08

3/3

Question 2c (3 marks)

Given that a shoe is defective, **find** the probability that it was made in Brazil.

1-0.4*0.95 0.62
=0.62

0/3

Question 3 (8 marks)

Consider the functions $f(x) = \frac{x-2}{3x-11}$, $x \neq \frac{11}{3}$ and $g(x) = x + 3, x \in \mathbb{R}$.

Question 3a (2 marks)

Show that $f \circ g(x) = \frac{x+1}{3x-2}$, $x \neq \frac{2}{3}$

$f \circ g(x) = \frac{x+1}{3x-2}$

$f(g(x)) = \frac{(x+3)-2}{3(x+3)-11}$

$= \frac{x+1}{3x+9-11}$

$= \frac{x+1}{3x-2}$

2/2

Question 3b (6 marks)

Find $(f \circ g)^{-1}(x)$, the inverse function of $f \circ g(x)$, and **write down** its domain.

$f \circ g(x) = \frac{x+1}{3x-2}$

$y = \frac{x+1}{3x-2}$

$x = \frac{y+1}{3y-2}$

$0 = \frac{y+1}{3y-2} - x$

3y-2=y+1-x

3y=y+1-x+2

2y=-x+3

$y = \frac{3-x}{2}$

$(f \circ g)^{-1}(x) = \frac{3-x}{2}$

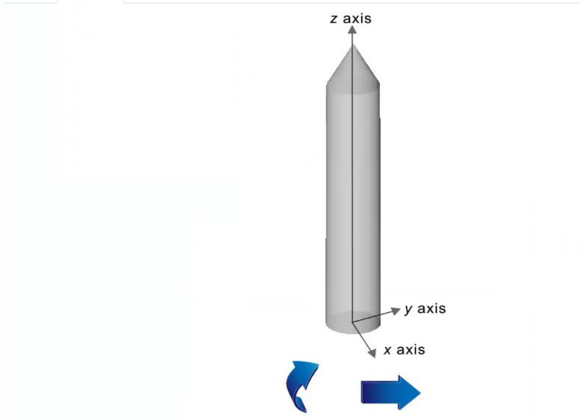
1/6 Wrong working.

Question 4 (9 marks)

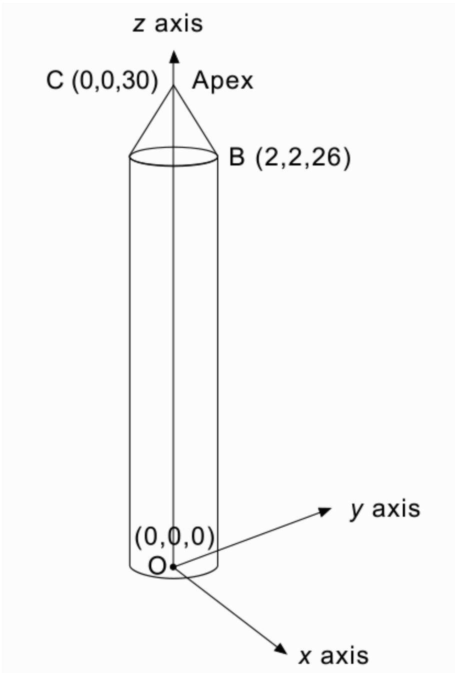
Here is information on Glendalough round tower in Ireland.

Video

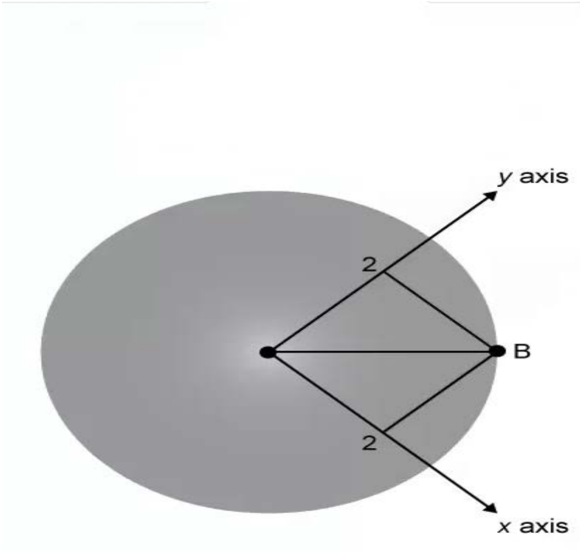
Simulator



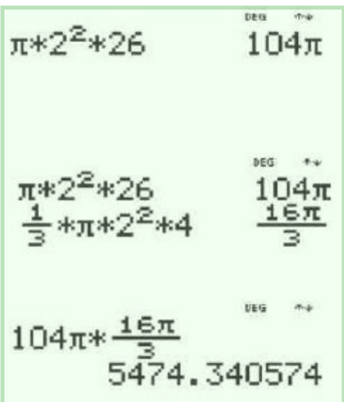
Tower dimensions



Tower cross section



The round tower is modelled as a cylinder and a cone. **Find** the total volume of the round tower. Express your answer as an exact value.



5/9

- .1 and .2 not awarded radius 2.

- .4 and .5 awarded substitutes and volume correct.

- .3, .6 and .7 awarded height cone correct, substitutes and volume correct.

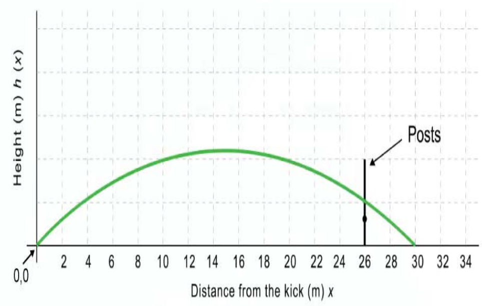
- .8 not awarded not add.

- .9 not awarded not exact response.

Question 5 (12 marks)

A ball is kicked from (0,0) towards the posts and describes the path of a parabola as shown in Tab 2.

A ball is kicked at the posts shown, the ball describes the path of a parabola.



The height in metres (m) of the ball above the ground is given by the function $h(x) = \frac{x(a-x)}{25}$.

Question 5a (2 marks)

Show that $a = 30$.

The two roots are 0 and 30
30-0=30 thus a=30

0/2 Not clear enough. What equation? Where does 30 - 0 = 30 come from?

Question 5b (2 marks)

Find the maximum height achieved by the ball.

when x=30/2=15 the ball reaches its peek.
when plugged in h(15)=9meters

2/2

To score points the ball is kicked from the point (0,0) and must go over a bar on the posts that is 3 m above the ground and 26 m away from the point the ball was kicked. This is illustrated in the diagram in Tab 2 above.
Remember the function for the height of the ball is $h(x) = \frac{x(a-x)}{25}$

Question 5c (3 marks)

Did the team score points? **Show** your working and reasoning.

When y=3 and x=26 and the function works then the teams should score points!
Plugged in:

$h(26) = \frac{26(30-26)}{25} = 4.16$m which is more than 3 thus the team scores

3/3

Question 5d (5 marks)

Find the total horizontal distance for which the ball is above 3 m.

When x= 2.75 and 27.25 the height is approximatley 3 thus the total distance between x1 and x2 for a height above 3 =24.5m

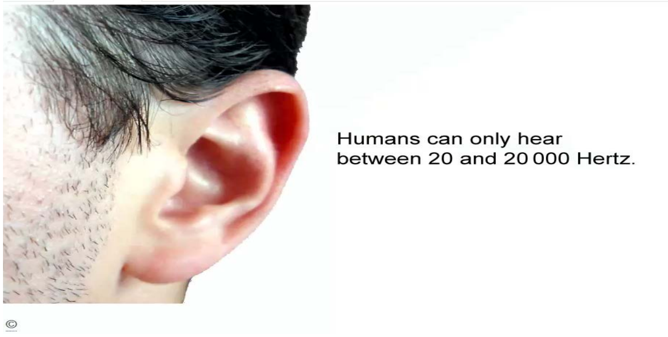
2/5 Estimates and subtracts for .4 and .5 ECF.

Question 6 (20 marks)

Music is a widely accepted form of artistic expression and creativity. In this question you will be presented with the instrument design of a piano.

Video

Hearing range



Here is a table with some information on the octaves and frequencies of the note A on the piano. The frequencies increase by a geometric progression.

Octave (n)	0	1	2	3	4	5	6	7
Note (A)	A_0	A_1	A_2	A_3	A_4	A_5	A_6	A_7
Frequency (F) Hertz (Hz)	27.5	55	110	220	440	880	?	?

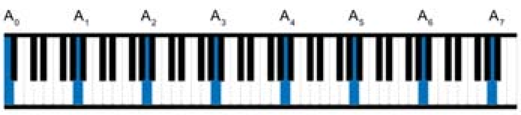
Question 6a (1 mark)

Write down what happens to the frequency F, as the octaves increase.

As the note changes upwards (A_0 to A_1 etc.), the frequency/Hertz of this note will increase by dubbling it's previous frequency (starting at A_0=27.5).

1/1 Doubling spelled wrongly.

Question 6b (2 marks)



Deduce that all the A notes on the piano are within human hearing.

A human can hear frequencies/hertz between 20 and 20.000. Note A_0 is the lowest A note on a normal piano, and 27.5. Hence clearly being within a human's hearing limit.

The highest A note a normal piano is A_7 and is reaches 3520 hertz. Hence both being at normal hearing frequencies.

2/2 Between is enough to indicate 20 < A < 20000.

Question 6c (2 marks)

Find a formula for F in terms of n.

$F_n=(A_{n-1})(2)$

A = note being calculated

0/2

Question 6d (3 marks)

Hence, rearrange your formula for part (c) to give n in terms of F.

$F_n=(A_{n-1})(2)$ becomes $N_f=(A_{f-1})(2)$

0/3

Question 6e (2 marks)

A note has frequency 28 160 Hz. Is it an A note? **Justify** your answer.

no becuase it is just above A_0, and hence not being an A note.

0/2

Question 6f (10 marks)

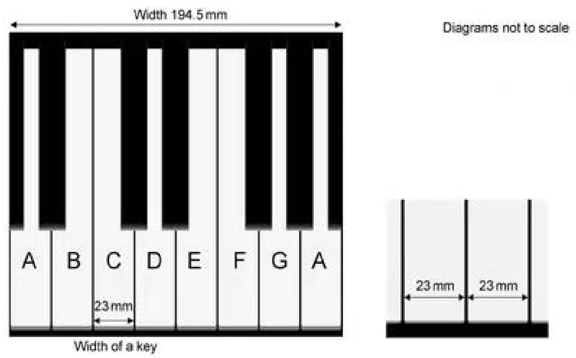
A new design for a piano will contain all of the A notes within human hearing.

Use the information provided in the tabs to **discuss** the practicality of the new design for the piano. In your discussion you should:

- identify the relevant information for the new design
- calculate relevant measurements for the new design
- justify your degree of accuracy
- justify the practicality of the new design.

Piano

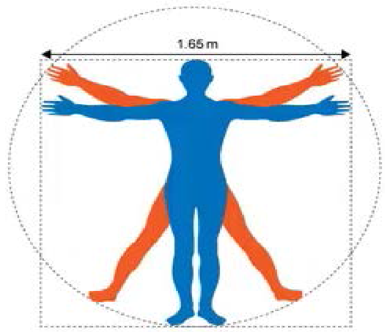
Keys



Hearing range

Humans can only hear between 20 and 20 000 Hertz.

Arm span



This new desing for pianos will practically work properly, and there are no real limitations to it. However, the width (A) is almost as long as the the arm span of a regular human body. And this may cuase discomfort, and may even hinder the ability to make proper music. However, there is a very important improvement to a normal piano, which is that all the A notes are now within human hearing abilities.

2/10

IR 0, CM 0, JD 0, PD says it is improvement and gives reason 1, QD limited 1

Candidate is talking about the existing piano.

Question 7 (18 marks)

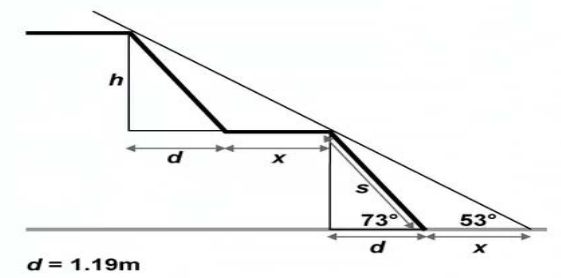
Ancient civilizations built pyramids to express cultural identity. In this question you will make calculations for a model of Chichén Iztá pyramid.

Chichén Iztá pyramid is composed of nine platforms. The width of the first platform is 55.3 m and on the ninth platform there is a temple that is 6 m high.

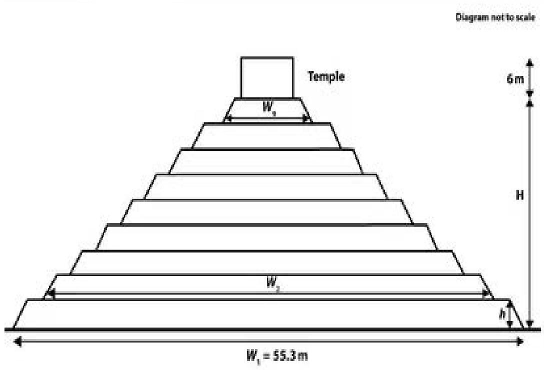
All lengths provided are given to the nearest centimetre (cm) and angles are to the nearest whole degree.

You are planning to construct a scale model of the Chichén Iztá Pyramid and will use 1:100 as scale for your model. In this question you will work out values of lengths. The validity of the accuracy of your model will be required in part (e).

Tab 1



Tab 2



Question 7a (2 marks)

Using the diagram in Tab 1, **show that** the sloping length *s* = 4.07 m to the nearest cm.

```
1.19÷cos(73)
        4.070161308
```

Hence aswer: 4.07 cm

1/2 .1 Not awarded. .2 For calculation.

Question 7b (2 marks)

Using the diagram in Tab 1, **find** the height *h* of each platform in your model.

Model height *h* 3.89 cm

```
sin(73)*4.07
        3.892160357
```

2/2

Question 7c (5 marks)

Using the diagram in Tab 2, **find** the width W2 for the second platform in your model.

Model width W_2 51.7 cm

0/5 Wrong answer, no working.

Question 7d (3 marks)

Using the diagram in Tab 2, **find** the width W_9 of the top platform in your model.

Model width W_9 14.2 cm

0/3 Wrong answer, no working.

Question 7e (6 marks)

Discuss the validity of your model for Chichén Iztá pyramid. In your discussion you should:

- identify relevant information required to discuss the validity
- consider the implications of your chosen degree of accuracy
- comment on the validity of the model to Chichén Iztá.

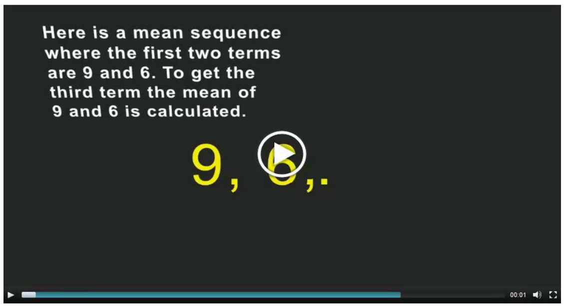
During the investigation above, the aswers were rounded off to the nearest cm. And hence, if those rounded off numbers are used in following questions, the answer will not be fully accurate anymore. And in this excersize, were you want to accurately calcute measurements of the pyramide, this can serve as an obstruction for an accurate investigation. Also, the model showing in diagrams above, cannot be fully accurate, this being becuase the temple must have been weathered or affected by any kind of force, the measurements cannot be so perfectly rounded off the way they are given. In addition, the model shown does not perfecty reflect the shown image, as the temple on the top is not very accurately visualised in the model. This is not a very important error, yet it can affect the investigation as it one requires a proper scale model for an investigation of such degree.

4/6 IR 2 measurements, temple. DA 1 nearest cm. CV 1 erosion.

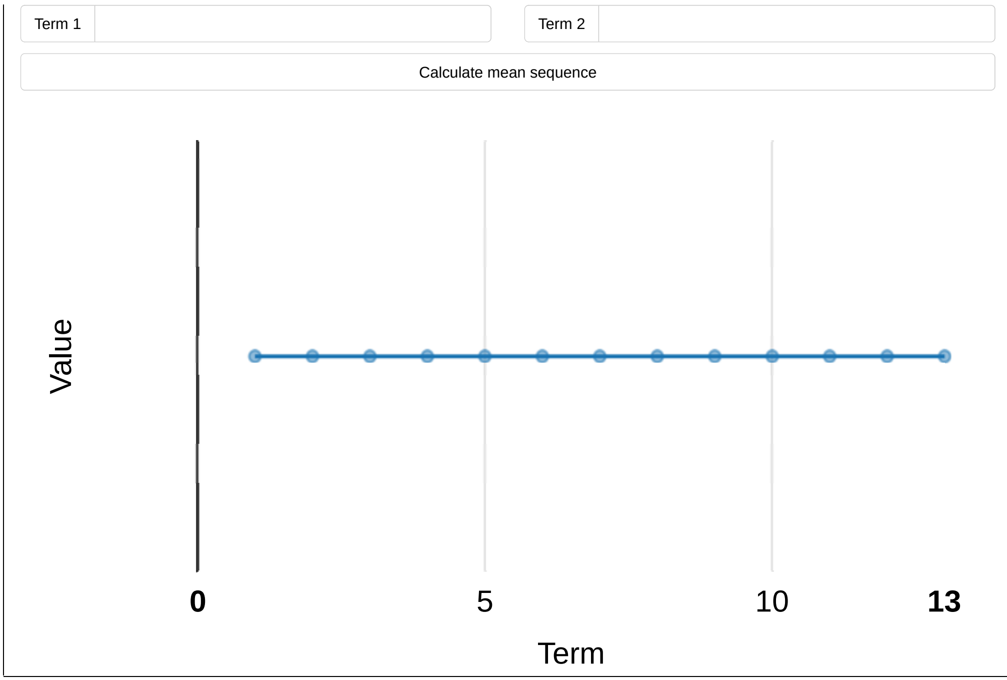
Question 8 (19 marks)

A recursive sequence is one where the next term is a specific combination of the previous terms.

In this question the recursive sequence is created where the next term is the mean of the previous two terms. This is called a mean sequence.



You can use the simulator to find the limit of a mean sequence. Input the first two terms of the sequence into the simulator and click 'Calculate mean sequence'.



Question 8a (1 mark)

Here is a mean sequence with starting terms 6 and 9.

6, 9, 7.5, 8.25, 7.875, ...

Write down the limit of the sequence.

8

1/1

All mean sequences have limits and there is a connection between the first two terms *a* and *b* and the limit *L* of each sequence.

Question 8b (2 marks)

The table below shows the first two terms and the limit *L*, for some mean sequences. The different starting terms *a* are provided and *b* is fixed at 3.

Write down the limit values *L*, for row 2, 3 and 4 in the table for the mean sequences provided. Use the simulator to help you.

Row	First term (a)	Second term (b)	Limit (L)
1	6	3	4
2	9	3	5
3	12	3	6
4	15	3	7

2/2

Question 8c (2 marks)

Describe in words **two** patterns you have found from the table.

As the first term increases by three and the second term stays the same the limit of the table increases by one.

2/2

Question 8d (2 marks)

Find a general rule connecting *a* and *L*.

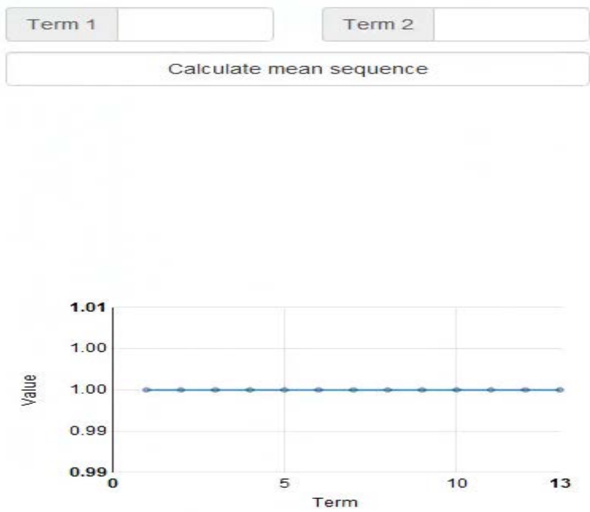
a-L=

0/2

Question 8e (12 marks)

Using your previous results, **investigate** and **find** a general rule connecting *a*, *b* and *L*. The simulator and a blank table is provided below to support your investigation. In your answer you should:

- describe patterns
- find a general rule
- test your general rule
- use correct mathematical notation
- notation prove or verify and justify your general rule.



First term (a)	Second term (b)	Limit (L)
1	2	1.67
3	4	3.67
5	6	5.67
7	8	7.67
9	10	9.67

In my investigation there are patterns which relate the first term and the limit. The pattern is that the limit follows the first term becaus e for example when the first term is 1 the limit is 1.67 and when the first term is 5 the limit is 5.67 therefore the limit increases from th e first term by 0.67.

The general rule for my investigation is:

L=a+0.67

Rule tested:

L= 9+0.67

= 9.67

Which is correct.

This rule has been proved through the testing of it.

4/12

New predictions.

New very simple rule which works here.

Not more than band 4-6, see note award 4.

Question 9 (20 marks)

We can also use an algebraic approach to investigate mean sequences.

If a and b are the first two terms, the mean sequence will start with the terms below.

$$a,\ b,\ \frac{1}{2}a + \frac{1}{2}b,\ \ldots$$

Question 9a (2 marks)

Show that the fourth term is $\frac{1}{4}a + \frac{3}{4}b$.

Term (T)	Sequence	Change in coefficient of *b*
1	a	
2	b	$+1$
3	$\frac{1}{2}a + \frac{1}{2}b$	$-\frac{1}{2}$
4	$\frac{1}{4}a + \frac{3}{4}b$	$+\frac{1}{4}$
5	$\frac{3}{8}a + \frac{5}{8}b$	$-\frac{1}{8}$
6	$\frac{5}{16}a + \frac{11}{16}b$	$+\frac{1}{16}$

$$\frac{b + \frac{1}{2}a + \frac{1}{2}b}{2} = \frac{1}{2}b + \frac{1}{4}a + \frac{1}{4}b$$

$$= \frac{1}{4}a + \frac{3}{4}b$$

2/2

Question 9b (4 marks)

Write down the values of x and y and explain how this series relates to the sequence in the table above.

Series

$1 = 1$

$1 - \frac{1}{2} = \frac{1}{2}$

$1 - \frac{1}{2} + \frac{1}{4} = x$

$1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} = y$

x = 1 - 1/2 + 1/4

1-0.5+0.25 0.75

y = 1 - 1/2 + 1/4 - 1/8 = 0.625

2/4

Question 9c (14 marks)

Investigate the algebraic approach further to **prove** or **verify** and **justify** the formula in question 8e.

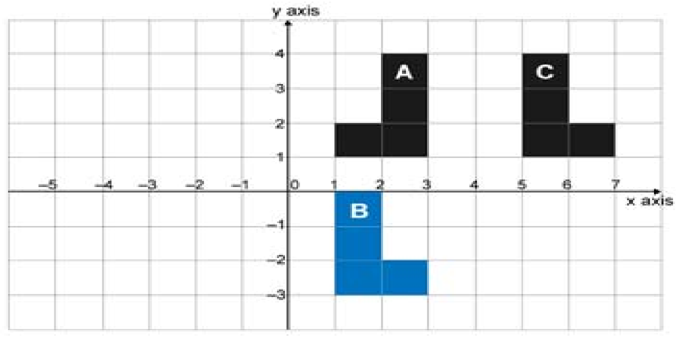
In your answer you should:

- describe patterns
- find one or more general rules
- test your general rule(s)
- use correct mathematical notation
- prove or verify and justify your general rule(s).

The pattern in this investigation is that the numbers subtract then add and it goes on but they subtract or add by half the previous number.

0/14

Question 1 (6 marks)



Question 1a (1 mark)

Write down the equation of the line that reflects shape A to shape C.

x=4

1/1

Question 1b (1 mark)

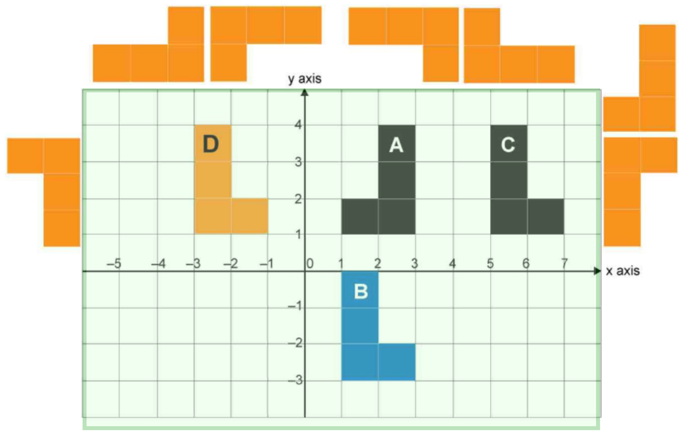
The vector ***c*** translates the shape B to shape C. **Write down** the vector ***c*** in the form $\begin{pmatrix} p \\ q \end{pmatrix}$.

$\begin{pmatrix} 4 \\ 4 \end{pmatrix}$

1/1

Question 1c (1 mark)

Translate shape B by ve $\left(-4 \right)$, **draw** and label the image D.



1/1

Question 1d (2 marks)

Find the scalar product ***c*** · ***d*** .

c d=4*(-4)+4*4=0

2/2

Question 1e (1 mark)

Explain the significance of your result in part (d).

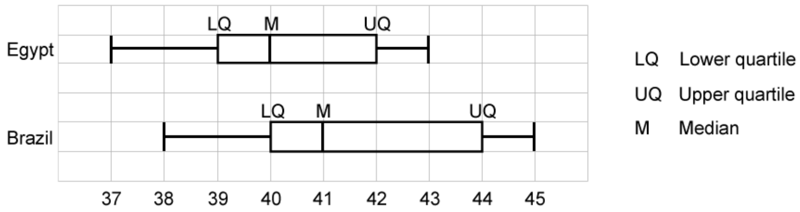
It means that

0/1

Question 2 (8 marks)

A manufacturer makes shoes to sell internationally. The manufacturer is contracted to make shoes for Egypt and Brazil.

The box plots show the sizes of shoes sold in Egypt and Brazil. The distributions shown in the box plots are based on the medians and quartiles of the shoe sizes sold.



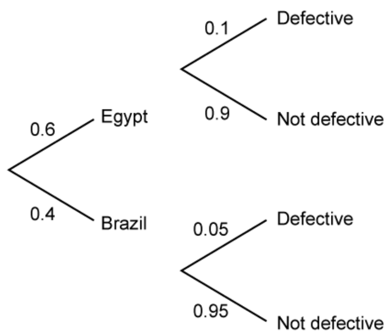
Question 2a (2 marks)

Compare the sizes of shoes sold in the two countries.

Overall Brazilian people have more bigger feet than people in Egypt. The median of Brazil is higher than Egypt - Brazil's median is 4 1 and the median of Egypt is 40.

1/2 Comment on median. Value of median. For 1 mark.

The manufacturer makes shoes in two factories; one in Egypt and one in Brazil. The factory in Egypt makes 60 % of the shoes and 10 % are defective. The factory in Brazil makes the rest of the shoes and 5 % are defective.



Question 2b (3 marks)

Find the probability that a shoe, chosen at random from either factory, is defective.

Probability:

$(0.6 \times 0.1) + (0.4 \times 0.05) \ = \ 0.08$

3/3

Question 2c (3 marks)

Given that a shoe is defective, **find** the probability that it was made in Brazil.

(0.4×0.05)

$= 0.02$

0/3 Formula not seen. Division not seen.

Question 3 (8 marks)

Consider the functions $f(x) = \frac{x-2}{3x-11}$, $x \neq \frac{11}{3}$ and $g(x) = x + 3, x \in \mathbb{R}$.

Question 3a (2 marks)

Show that $f \circ g(x) = \frac{x+1}{3x-2}$, $x \neq \frac{2}{3}$

$$f(g(x)) = \frac{(x+3)-2}{3(x+3)-11}$$
$$f(g(x)) = \frac{x+3-2}{3x+9-11}$$
$$f(g(x)) = \frac{x+1}{3x-2}$$

2/2 .1 Substitutes. .2 Brackets off.

Question 3b (6 marks)

Find $(f \circ g)^{-1}(x)$, the inverse function of $f \circ g(x)$, and **write down** its domain.

$$y = \frac{x+1}{3x-2}$$
$$x = \frac{y+1}{3y-2}$$
$$y + 1 = x(3y-2)$$
$$y = 3xy - 2x - 1$$

$f(g(x))^{-1} = 3xy - 2x - 1$

$x \in \mathbb{R}$

Domain =

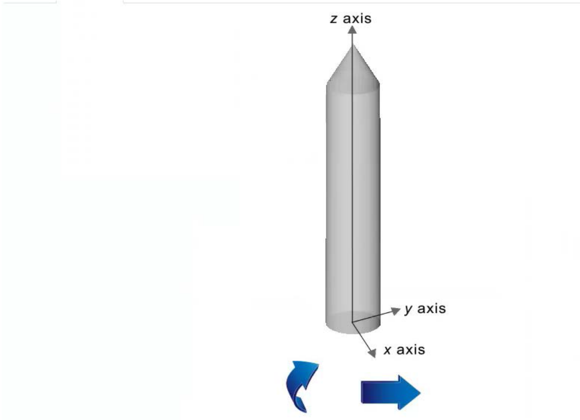
2/6
.5 exchange x and y.
.1 to y and cross multiply.
.3 and .4 not awarded - no common factor; does not write y in terms of x.
No further marks.

Question 4 (9 marks)

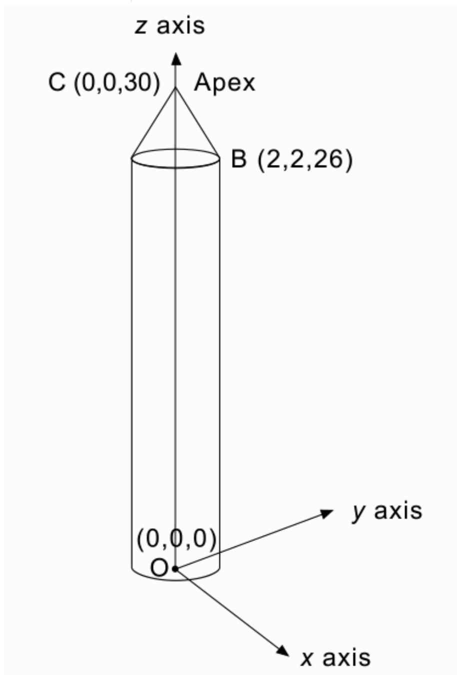
Here is information on Glendalough round tower in Ireland.

Video

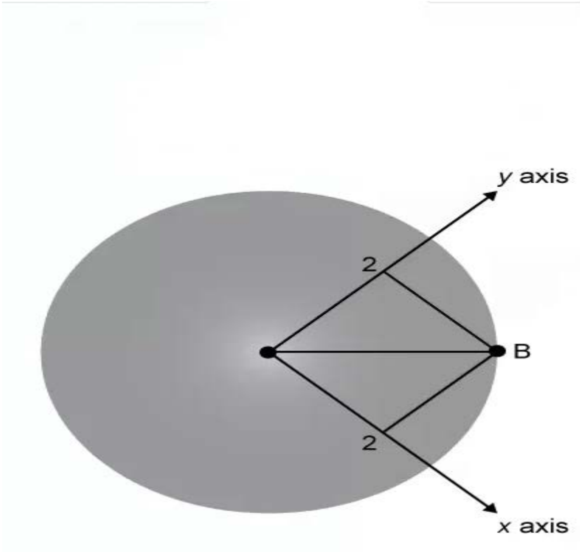
Simulator



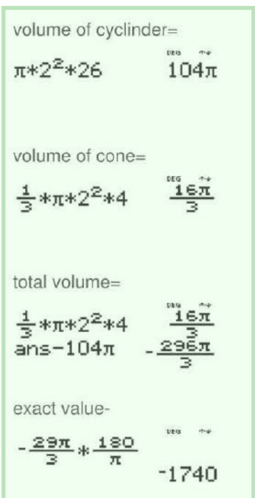
Tower dimensions



Tower cross section



The round tower is modelled as a cylinder and a cone. **Find** the total volume of the round tower. Express your answer as an exact value.



5/9

.1 and .2 not awarded radius 2.

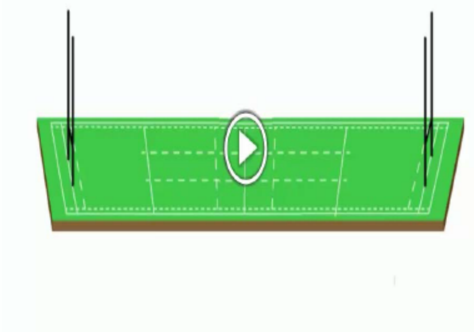
.6 and .7 awarded substitutes and correct volume.

.3 and .4 and .5 awarded height, substitutes and correct volume.

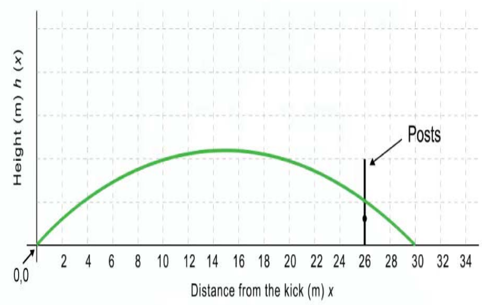
.8 not awarded not add.

Question 5 (12 marks)

A ball is kicked from (0,0) towards the posts and describes the path of a parabola as shown in Tab 2.



A ball is kicked at the posts shown, the ball describes the path of a parabola.



The height in metres (m) of the ball above the ground is given by the function $h(x) = \dfrac{x(a-x)}{25}$.

Question 5a (2 marks)

Show that $a = 30$.

The height of the ball at 30 m is 0. Therefore $h(30) = \dfrac{30(a-30)}{25}$

h(30) = 0

0 x 25 = 30a - 900

30a = 900

a = 900/30 = 30

so a = 30

2/2
.1 Substitutes.
.2 finds a.

Question 5b (2 marks)

Find the maximum height achieved by the ball.

h(15) = 15(30 - 15)/25

h(15) = 225/25 = 9 metres

2/2

To score points the ball is kicked from the point (0,0) and must go over a bar on the posts that is 3 m above the ground and 26 m away from the point the ball was kicked. This is illustrated in the diagram in Tab 2 above.

Remember the function for the height of the ball is $h(x) = \dfrac{x(a-x)}{25}$

Question 5c (3 marks)

Did the team score points? **Show** your working and reasoning.

Yes, the team did score points because

h(x) = 26 x 4/25 = 4.16 metres. The ball went above the post's limit.

2/3
.1 and .2 awarded but does not state 4.16 greated than 3.
.3 not awarded.

Question 5d (5 marks)

Find the total horizontal distance for which the ball is above 3 m.

$3 = \dfrac{x(30-x)}{25}$

$75 = 30x - x^2$

$75 - 30x + x^2 = 0$

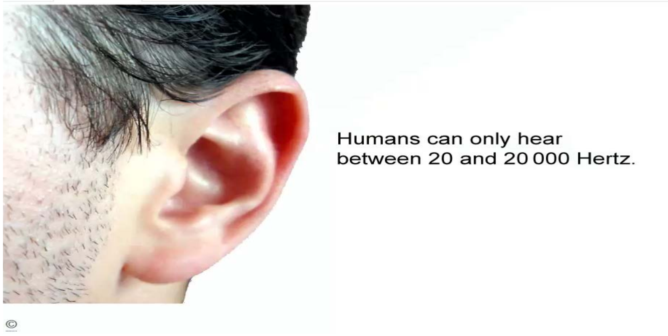
2/5
.1 sets up equation
.2 cross multiplies.

Question 6 (20 marks)

Music is a widely accepted form of artistic expression and creativity. In this question you will be presented with the instrument design of a piano.

Video

Hearing range



Here is a table with some information on the octaves and frequencies of the note A on the piano. The frequencies increase by a geometric progression.

Octave (n)	0	1	2	3	4	5	6	7
Note (A)	A_0	A_1	A_2	A_3	A_4	A_5	A_6	A_7
Frequency (F) Hertz (Hz)	27.5	55	110	220	440	880	?	?

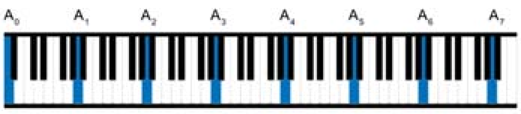
Question 6a (1 mark)

Write down what happens to the frequency F, as the octaves increase.

The frequency increases as the octaves increase.

0/1

Question 6b (2 marks)



Deduce that all the A notes on the piano are within human hearing.

According to the table above, the frequency from A_0 to A_7 has the range of 27.5 to __. The human hearing ranges from 20 to 20000 hertz, so the A notes are within that range of human hearing.

0/2

Question 6c (2 marks)

Find a formula for F in terms of n.

The geometric sequence formula works because every consecutive number in the sequence of the frequencies have a ratio of 2. ($F_n = F_1r^{n-1}$)

2/2 Awarded for fully correct formula seen in e).

Question 6d (3 marks)

Hence, rearrange your formula for part (c) to give n in terms of F.

$F_n = F_1 r^{n-1}$

Example - if finding for frequency of the second octave

$F_2 = 55 \times 2^{2-1}$

$= 55 \times 2^1$

$= 55 \times 2 = 110$

The formula works because the second octave has a frequency of 110 hertz as shown in the table.

0/3

Question 6e (2 marks)

A note has frequency 28 160 Hz. Is it an A note? **Justify** your answer.

The note with the frequency of 28 160 Hz is an A note because the frequency value works for the geometrical sequence formula that was established in 6d: $F_n = F_1 r^{n-1}$. The nth term using this formula found that 28160 hertz is the frequency of the A $_{10}$.

Working backwards using the formula:

$F_{n=u_1 r^{n-1}}$

$28160 = 55 \times 2^{n-1}$

$\frac{28160}{55} = 2^{n-1}$

$512 = 2^{n-1}$

$2^9 = 2^{n-1}$

Indices

$9 = n - 1$

$10 = n$

Therefore, the octave (n) of the A note labelled as A_n that has the frequency of 28160 Hz is 10. This can be defined as an A note bec ause the frequency of 28160 worked in the geometric sequence formula in order to establish the final result of 10 as its octave. This A note can be labelled as A10.

2/2 Award in part c for fully correct formula seen here.

Question 6f (10 marks)

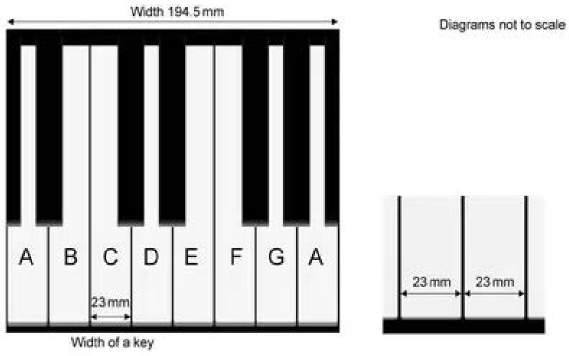
A new design for a piano will contain all of the A notes within human hearing.

Use the information provided in the tabs to **discuss** the practicality of the new design for the piano. In your discussion you should:

- identify the relevant information for the new design
- calculate relevant measurements for the new design
- justify your degree of accuracy
- justify the practicality of the new design.

Piano

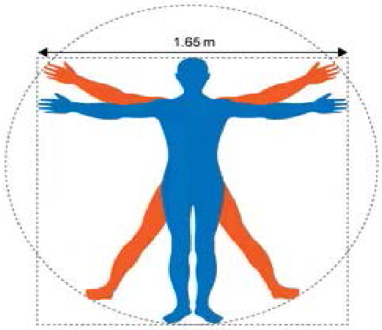
Keys



Hearing range

Humans can only hear between 20 and 20 000 Hertz.

Arm span



The practicality of the new design is suitable for easier use and definition of the a person because the capacity of our human hearing and arm span is enough to cover all the notes in the keyboard and hear them. The information needed to find this out is the total length of the keys and how many octaves of the A notes can the piano hold, arm span to length of keys ratio. All measurements can be converted into millimeters.

Measurements:

Total Width = 1480 mm

Total width of 1 A octave set = 161 mm

Total width of 7 octave set = 1127

Arm span = 1650 mm

The maximum number of octaves for an entire set of notes from A-G is 9. Each note has a length of 23 mm. There are 7 notes in one set (A-G), so that is 23 x 7 = 161 mm. Since one set of notes is 161, if there were 9 sets of notes, the total length occupied by the keys would be 1449. Moreover, the human hearing can only hear until the octave of 9 because using the formula, the frequency would result to 14080. This is the maximun frequency we can hear. However, there are still spaces on the side so less sets of keys are needed. It would be ideal that there should be only 7 or maximum 8 sets of octaves on the keyboard. The design also satisfies the arm span because an average arm length (in total from left to right + torso) is 1650 mm. There will be enough arm length and flexibility to reach different ends of the keyboard.

The degree of accuracy use is millimeters because it is easier to find the differences between the total length of the keyboard against the arm span and the total length of the piano. If the measurements were used in the unit of centimeters, the numbers would be too small as the results will be in decimals. This provides less accuracy and precision with the exact measurements of each part of the piano. The smallest measurement is one key which is measured as 23 mm; this is too small to convert into centimeters as it would still have a decimal.

7/10

IR several seen 2 marks.

CM mentions gaps but does not deal with it 1 mark.

JD mentiones it but misunderstands.

PD justifies with numbers.

QD balanced 2 marks.

Question 7 (18 marks)

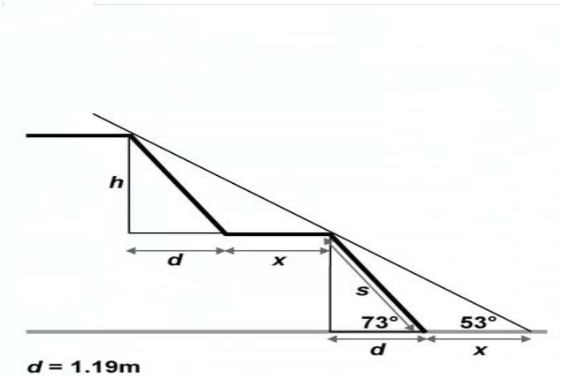
Ancient civilizations built pyramids to express cultural identity. In this question you will make calculations for a model of Chichén Iztá pyramid.

Chichén Iztá pyramid is composed of nine platforms. The width of the first platform is 55.3 m and on the ninth platform there is a temple that is 6 m high.

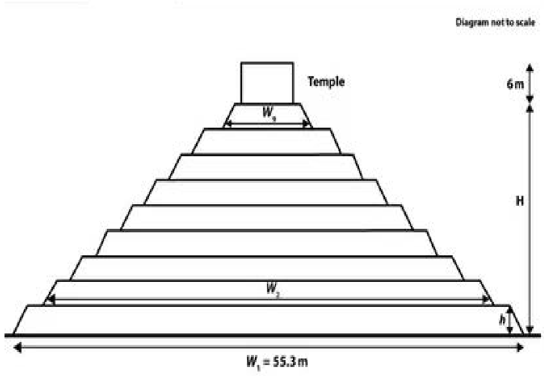
All lengths provided are given to the nearest centimetre (cm) and angles are to the nearest whole degree.

You are planning to construct a scale model of the Chichén Iztá Pyramid and will use 1:100 as scale for your model. In this question you will work out values of lengths. The validity of the accuracy of your model will be required in part (e).

Tab 1



Tab 2



Question 7a (2 marks)

Using the diagram in Tab 1, **show that** the sloping length *s* = 4.07 m to the nearest cm.

$\sin(x) = \frac{1.19}{4.07}$

$\sin(x) =$

1.19÷4.07
0.292383292

$\sin^{-1}(0.29)=$

1.19÷4.07
0.292383292
sin⁻¹(ans)
17.00069426

theefore we may prove that s=4.07 because we can also whoe that the top right angle is 17 degrees by saying 180-(73+90)= 17

2/2 Using sin (17) but in reverse. Valid alternative.

Question 7b (2 marks)

Using the diagram in Tab 1, **find** the height *h* of each platform in your model.

Model height *h* 151.5 cm

$4.07^2 - 1.19^2 = 15.15$m

1/2 Forgets to take square root.

Question 7c (5 marks)

Using the diagram in Tab 2, **find** the width W2 for the second platform in your model.

Model width W_2 529.2 cm

55.3 - 2(1.19)=

55.3-2*(1.19)
52.92

1/5
.4 not awarded, must be for their value plus 1.19.
.5 ECF.

Question 7d (3 marks)

Using the diagram in Tab 2, **find** the width W_9 of the top platform in your model.
Model width W_9 362.6 cm

55.3- 16(1.19)=36.26

3/3 ECF for continued use of 1.19.

Question 7e (6 marks)

Discuss the validity of your model for Chichén Iztá pyramid. In your discussion you should:

- identify relevant information required to discuss the validity
- consider the implications of your chosen degree of accuracy
- comment on the validity of the model to Chichén Iztá.

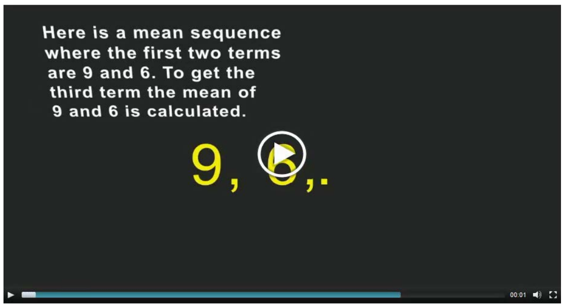
Although we may be able to look at a pyramid like this and assume that the edges are sharp and exact just like in the diagrams made , but in the end the pyramid was made by hand therefore measurements such as the angles and measurements of lengths x are hard to define and change at each layer of the pyramid. The results gained in the above investigation do give an approximation and clear i dea of what it ight have been when it was first built but also then I do not believe realitically it was so precise. With the numbers ued a bove nothing was rounded before the end and when rounded at east one decimal place was kept. Other things mayhave affected the measuring of the pyramid as there are many patterns and wind and anturl forces may have worn it down a lot. The model and diagra m used are both effective in giving a clear idea aboutwhat the building looks like and how it was built.

3/6
IR 1 lengths.
DA 1 rounding and decimal places.
CV 1.

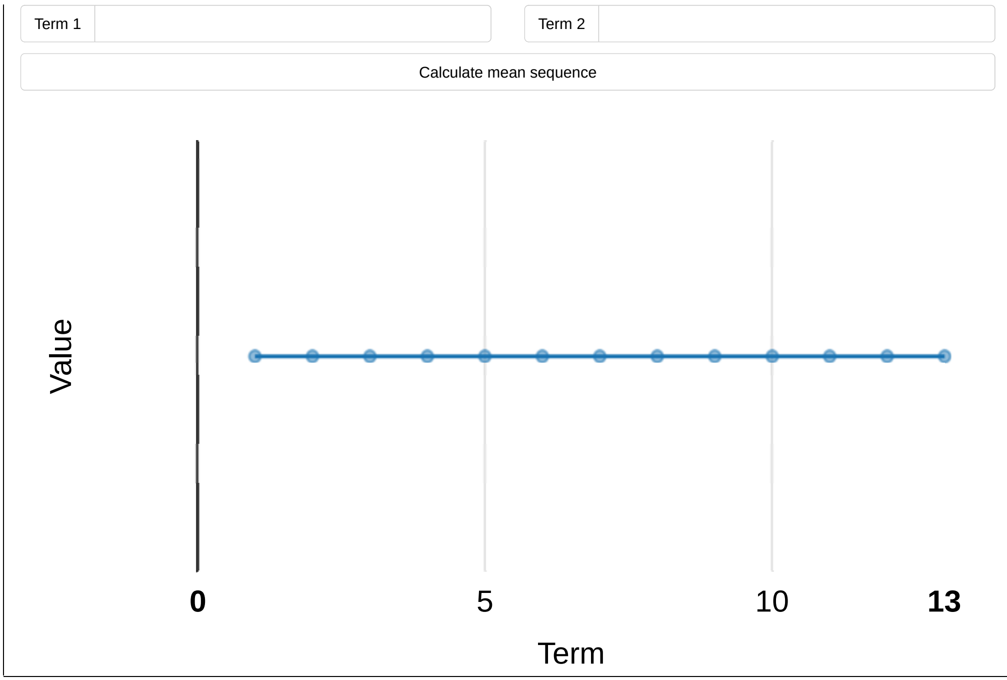
Question 8 (19 marks)

A recursive sequence is one where the next term is a specific combination of the previous terms.

In this question the recursive sequence is created where the next term is the mean of the previous two terms. This is called a mean sequence.



You can use the simulator to find the limit of a mean sequence. Input the first two terms of the sequence into the simulator and click 'Calculate mean sequence'.



Question 8a (1 mark)

Here is a mean sequence with starting terms 6 and 9.

6, 9, 7.5, 8.25, 7.875, ...

Write down the limit of the sequence.

7

0/1

All mean sequences have limits and there is a connection between the first two terms *a* and *b* and the limit *L* of each sequence.

Question 8b (2 marks)

The table below shows the first two terms and the limit *L*, for some mean sequences. The different starting terms *a* are provided and *b* is fixed at 3.

Write down the limit values *L*, for row 2, 3 and 4 in the table for the mean sequences provided. Use the simulator to help you.

Row	First term (a)	Second term (b)	Limit (L)
1	6	3	4
2	9	3	5
3	12	3	6
4	15	3	7

2/2

Question 8c (2 marks)

Describe in words **two** patterns you have found from the table.

$a=a_{n-1}+3$

$b=b$

$c=c_{n-1}+1$

2/2 Not in words but is correct. No penalty.

Question 8d (2 marks)

Find a general rule connecting *a* and *L*.

6/3+2=4

9/3+2=5

12/3+2=6

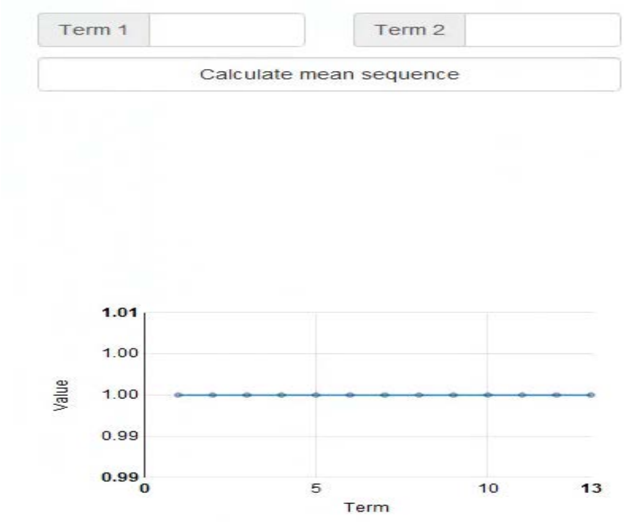
a/b+2=L

2/2

Question 8e (12 marks)

Using your previous results, **investigate** and **find** a general rule connecting *a*, *b* and *L*. The simulator and a blank table is provided below to support your investigation. In your answer you should:

- describe patterns
- find a general rule
- test your general rule
- use correct mathematical notation
- notation prove or verify and justify your general rule.



-

First term (a)	Second term (b)	Limit (L)
3	6	5
3	9	7
3	12	9

when the term1 equal to 3 and term2 equal 3x, then the b/a*2+1=c can been use.

6/3*2+1=5

9/3*2+1=7

12/3*2+1=9...

9/12
Uses a, b, c as a,b, L
Rule is correct. Not consistent with part (d). So consistent with some findings. Fits band 7-9
Rule is tested
Award 9 maximum of the band

Question 9 (20 marks)

We can also use an algebraic approach to investigate mean sequences.

If *a* and *b* are the first two terms, the mean sequence will start with the terms below.

$$a,\ b,\ \frac{1}{2}a + \frac{1}{2}b,\ \ldots$$

Question 9a (2 marks)

Show that the fourth term is $\frac{1}{4}a + \frac{3}{4}b$.

Term (T)	Sequence	Change in coefficient of *b*
1	a	
2	b	$+1$
3	$\frac{1}{2}a + \frac{1}{2}b$	$-\frac{1}{2}$
4	$\frac{1}{4}a + \frac{3}{4}b$	$+\frac{1}{4}$
5	$\frac{3}{8}a + \frac{5}{8}b$	$-\frac{1}{8}$
6	$\frac{5}{16}a + \frac{11}{16}b$	$+\frac{1}{16}$

+a

-a+b

+1/2a-1/2b

-1/4a+1/4b

a-a+b+1/2a-1/2b+1/4b-1/4a=1/4a+3/4b

0/2 Appears to use own rule based on adding and subtracting. Not acceptable, the rule given is based on means. Candidate has shown why this rule would work based on just three lines of the table.

Question 9b (4 marks)

Write down the values of *x* and *y* and explain how this series relates to the sequence in the table above.

Series

$1 = 1$

$1 - \frac{1}{2} = \frac{1}{2}$

$1 - \frac{1}{2} + \frac{1}{4} = x$

$1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} = y$

x=3/4

y=5/8

$u_1(1-r^n)/1-r$

r=-1/2

$1(1-(-0.5)^n)/1.5$

3/4 Formula correctly finds coefficient of b in terms of n. Only if n starts at row 2 of the table ie with b.
The sequence is implied for .1 the link to the sequence is not explained.
.4 not awarded.

Question 9c (14 marks)

Investigate the algebraic approach further to **prove** or **verify** and **justify** the formula in question 8e. In your answer you should:

- describe patterns
- find one or more general rules
- test your general rule(s)
- use correct mathematical notation
- prove or verify and justify your general rule(s).

b/a*2+1=c(b=xa,x∈U,a=3)

on this rule, the a have to be 3 and b equals to x times of a

and when we take some numbers into this rule, we can see, that rule is correct, for example:

6 is 2 times of 3 and when it plus 1 it will equals to the limit:5

9 is 3 times of 3 when it plus 1 it will equals to the limit:7...

3/14

Examines further their rule for 8 e.

Does not examine the algebraic approach further.

Does not describe patterns relating to algebraic approach.

Test the rule.

Gets 3 in band 3-5, see note.